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# Nontrivial bookkeeping: a mechanical perspective

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## Abstract

Although economic practice is measured by bookkeeping, economic theory neglects its role. We want to bridge this gap. We analyze the structure of bookkeeping from a mechanical viewpoint and compare money with momentum. Bookkeeping and income statements translate to the physicist vocabulary of momentum, energy and force. Financial transactions become Feynman-graphs. Although the realization principle enforces conservation of momentum in each currency, no conservation of energy is implemented by bookkeeping. We discuss the transaction axiomatics of bookkeeping from this physical viewpoint.

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## 1. Introduction

Economic theory is reluctant to give a measurement theory of its variables. Even central banks like the American FED and its Swiss counterpart recently gave up to define the quantity of money. Yet from a physical viewpoint, any variable in a model has to be accompanied by a measurement protocol. Although bookkeeping does serve as the measurement method for economy in practice [1–8], economic theory is reluctant to apply or discuss it. Econophysics might bridge this gap. We explore an analogy which connects bookkeeping with the physical methods of mechanics. In doing so, we find an intricate interplay of monetary transfer and exchange with the creation and annihilation of monetary units.

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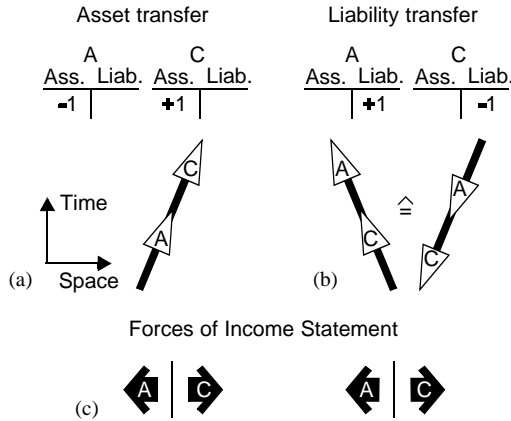


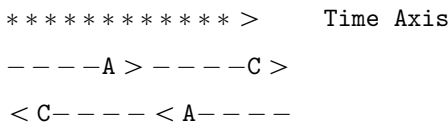
Fig. 1. Bookkeeping mechanics translates bookkeeping to Feynman-graphs. (a) Asset currency units are translated to positive momentum of particles, shown in the space–time of Feynman-graphs. Liabilities likewise with negative momentum. An asset transfer from A to C translates to a bouncing of asset particle momentum. Valueless resting particles are not shown. (b) The same amount can be transferred with a liability transfer from C to A. Since liability momentum from the past is equal to asset momentum from the future, we can also show it as a time reversed asset transfer. In quantum electrodynamics, a positron cannot be distinguished from its antiparticle electron moving backwards in time. Likewise, a liability into the future is identical to an asset moving from the future since momentum is identical when space and time is inverted. This idea coincides with economic wisdom: asset is value originating from the past and liability is value to be expected from the future. (c) The income statement translates to a force balance, extracted from the graphs as time derivative over finite time spans. Due to momentum conservation an income statement balances expense force to the left with revenue force to the right.

## 2. Mechanics of bookkeeping

The analogy was introduced in parts before [9]. Bookkeeping is based on a scheme of adding and subtracting between assets and its negative counterpart liabilities. We translate it to the physical adding scheme of momentum. Consider equally massed particles moving along one dimension. We will transcribe asset currency units to positive momentum to the right and liability currency units to negative momentum to the left.

As the particles bounce, the conservation of momentum ensures that momentum given by a particle will be received by other particles. Particle collision therefore models the transfer of money. We show two simple transfers in Fig. 1 in the space–time of a Feynman-graph with space to the left and time to the top [10].

Quite often, discussions are text based and exchanging Feynman-graphs by e-mail is cumbersome. A text-based representation is obtained with a coarse space visualization to make particles move virtually vertical and then rotating the graph by 90° clockwise. The two transfers of Fig. 1 become



Note that the liability transfer is performed at a later time. One can add the number of currency units into the arrows and mark currencies with different arrow characters, for example by choosing ‘- - - -’ vs. ‘====’.

Bookkeeping uses an income statement (Fig. 1c) to measure the profit of an agent over a given time span. Profit increases if assets increase or liabilities decrease, likewise it decreases if assets decrease or liabilities increase. In the mechanical picture, we derive the same result by calculating the momentum change over time for the particles of an agent. Since momentum change over time is the physical definition of a force, the profit of an income statement is derived from the forces which accelerate and decelerate the particles of an agent over a given time span. We therefore see that the income statement is the time derivative of assets and liabilities. Income statement and bookkeeping are the two representations of double entry bookkeeping.

### 3. Axiomatics of bookkeeping

In the following, we decompose bookkeeping into its axiomatic structure. To do so, we group transactions with an identical income statement force. We begin with a money transfer from A to C, shown in Fig. 2a. Four different Feynman-graphs can implement the transfer. In addition to the transfer of asset (2) and transfer of liability (3) discussed before in Fig. 1, we find pair creation (1) and pair annihilation (4) of assets and liabilities. In a transfer by creation, A increases its liability and C increases its assets. In a transfer by annihilation, A decreases its asset and C decreases its liability. The income statement of all four transfers are identical: the particles of A lose momentum and the particles of C gain momentum. For a given transfer the initial conditions of A and C will decide, which of the four Feynman-graphs will be chosen. For example if A has no assets and C has assets, a transfer by creation will be used (Fig. 2a, (1)).

From the physical perspective we see that currencies exist as an intertemporal loop between creation and annihilation. They connect assets from the past with assets from the future. Money exists in time intervals.

All the transfers and exchanges are realized between agents. Therefore, currency units which are given by an agent are received by another agent in an ideal monetary memory. Thus the income statement forces of all agents have to balance to zero: *actio* is *reactio*. This is an important statement in a multi-currency environment, since *actio* can only be balanced with the *reactio* of the same currency. Interactions between currencies have to be realized by exchanges. The result is conservation of momentum for each currency.

On the other hand, conservation of energy is not implemented in bookkeeping. For example, in a transfer by creation, the total momentum is constant, but the total energy has increased above the zero value before. We can give a microscopic definition of the quantity of money by counting the absolute number of asset and liability units. This is more general than the economists definition counting only certain bank liabilities [3–8].

More pair creations than pair annihilations will increase the quantity of money in a specific currency and therefore increase its price level on the market; we experience an

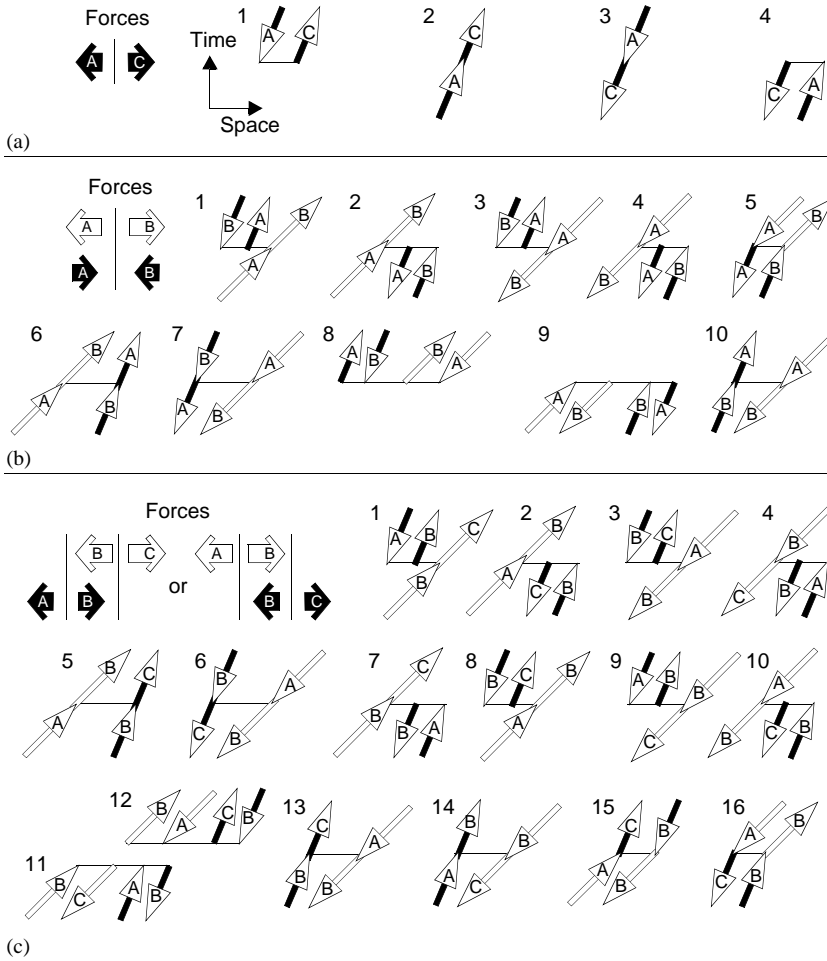


Fig. 2. Axiomatics of bookkeeping. We show all possible transactions in bookkeeping of (a) money transfer from A to C, (b) exchange between two currencies between A and B and (c) of money transfer from A to C through a mediator B. The income statement forces generate in each case a multitude of 4,10 and 16 transaction possibilities. They are selected depending on the initial conditions of assets and liabilities of the participating agents.

inflation. By analyzing inflationary scenarios using the bookkeeping of Feynman-graphs, we might gain a more precise analytical tool. An energy conservation of money should be understood from a bookkeeping perspective.

We consider currency exchange between two agents. A wants to obtain black currency instead of white currency and has found an agent B with opposite needs. The income statement forces are shown with the possible Feynman-graphs of the exchange in Fig. 2b. We have assumed an exchange rate of 1:2. A obtains 1 black currency unit and gives away 2 white currency units. B increases its momentum by 2 white

currency units and decreases it by 1 black currency unit. The momentum is conserved in each currency. From this income statement, we derive 10 different Feynman graphs (Fig. 2b). Only one of them (6) is a direct asset exchange. We can also exchange by: (1) asset-assisted creation, (2) asset-assisted annihilation, (3) liability-assisted creation, (4) liability-assisted annihilation, (5) exchange by recreation, (6) exchange of assets, (7) exchange of liabilities, (8) exchange by double creation, (9) exchange by double annihilation, and (10) exchange of asset versus liability.

Textbooks of double entry bookkeeping skip over this structure since they discuss bookkeeping at the time derivative level of income statements. They only show changes in asset and liability and do not discuss the interplay between initial conditions and the bookkeeping. Very often the exchange between currencies is totally neglected since standard bookkeeping does not allow multiple currencies in the same bookkeeping. Yet, in contrast to literature, bookkeepers at banks very well know the four transfer modes of Fig. 2a.

The case of a bank becomes quite interesting from this physical viewpoint. We approach banking from the viewpoint of money transfer. The possibilities to transfer money from A to C through a bank B is given in Fig. 2c. Our general approach allows the usage of two currencies. Agent A gives away in white and agent C obtains in black. The bank B has to provide the exchange between black and white. Likewise, agent A gives away in black and agent C obtains in white with an inverted exchange for B. Like before, we show the income statement force and the graphs for this transfer. We find 16 money transfer graphs. Naively, we expected only one of them: in case (5) assets are transferred from A to B in white and from B to C in black. But from different initial conditions, we might even apply double creations (12) or annihilations (11) and increase or decrease the total energy. Further discussions of banking on the basis of this two-currency picture of bookkeeping is out of the scope of this paper. First evaluations show an interesting dynamics due to the floating exchange rate between the two currencies [11].

To conclude, we translate the axiomatics of bookkeeping to the mechanics of particle collision. Financial transactions become Feynman-graphs. This approach is called bookkeeping mechanics and connects bookkeeping reality with statistical mechanics of particles [12].

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